

The rigid symmetries of bosonic D-strings

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We analyse the classical symmetries of bosonic D-string actions and generalizations thereof. Among others, we show that the simplest actions of this type have infinitely many nontrivial rigid symmetries which act nontrivially and nonlinearly both on the target space coordinates and on the $U(1)$ gauge field, and form a Kač-Moody version of the Weyl algebra (= Poincaré algebra + dilatations).

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INTRODUCTION

Much progress has been made lately in constructing κ -invariant actions for D - p -branes [1–3], generalizing earlier work [4]. Typically, the “bosonic part” of these actions is of the Born-Infeld type, such as

$$\begin{aligned} S_p &= \int d^{p+1}\sigma \sqrt{|\det(\mathcal{G}_{\mu\nu} + F_{\mu\nu})|}, \\ \mathcal{G}_{\mu\nu} &= \eta_{mn}\partial_\mu x^m \cdot \partial_\nu x^n, \\ F_{\mu\nu} &= \partial_\mu a_\nu - \partial_\nu a_\mu. \end{aligned} \quad (1)$$

Here, the x^m are target space coordinates, η_{mn} is a flat target space metric, and a_μ is an abelian gauge field living in the world volume.

Important properties of actions are of course their symmetries. In particular one may ask: What are the rigid and gauge symmetries of (1)? To what degree is this action determined by symmetries alone? In this letter we analyse these questions for the case $p = 1$. Our results apply not only to the action (1) but also to generalizations thereof which will be given below. We obtained these results by an analysis of the BRST cohomology which will be given in [5].

Although the cohomological analysis parallels quite closely the one for bosonic strings carried out in [6], the results are surprisingly rather different. For instance, while the usual bosonic string in a flat target space has only finitely many rigid symmetries before gauge fixing [6], we will show that, for $p = 1$, the action (1) has *infinitely many* nontrivial rigid symmetries. Among them there are of course the obvious Poincaré symmetries which reflect the isometries of the target space and coincide with the rigid symmetries of the bosonic string. However we find also previously unnoticed rigid symmetries which are nonlinearly realized and transform both the x^m and the gauge field a_μ . Together with the familiar Poincaré symmetries, the new symmetries form a

Kač-Moody version of the Weyl algebra (= Poincaré algebra + dilatations). We stress that these symmetries are present already *before* fixing a gauge. They should therefore not be confused with the Kač-Moody symmetries of sigma models discussed e.g. in [7] as the latter emerge just as residual symmetries of Weyl and diffeomorphism invariant actions in suitable gauges.

The fact that the new symmetries transform a_μ nontrivially has remarkable consequences. In particular there are symmetry transformations which map solutions of the equations of motion with trivial gauge field (zero or pure gauge) to other solutions with nonvanishing field strength $F_{\mu\nu}$, and thus usual bosonic strings to D -strings. As the field strength contributes to the string tension [8], the new symmetries therefore also relate strings with different tension and might thus be viewed as “stringy symmetries”. It is striking that these properties of the new symmetries are similar to those of dualities [9,10] relating bosonic and D -strings. One may speculate whether the new symmetries reflect part of the symmetry structure of an underlying (“M”) theory. As they are nonlinearly realized, one might for instance suspect that they emerge somehow as broken symmetries of that theory.

ACTIONS

To motivate and explain our approach, we note that (1) can be cast in a more convenient form [11,12] with Lagrangian

$$L_p = \frac{1}{2} \sqrt{\varrho} [\varrho^{\mu\nu} (\mathcal{G}_{\mu\nu} + F_{\mu\nu}) - (p-1)] \quad (2)$$

where $\varrho = |\det(\varrho_{\mu\nu})|$. In this formulation, the $\varrho_{\mu\nu}$ are auxiliary fields ($\varrho^{\mu\nu}$ denotes the inverse of $\varrho_{\mu\nu}$). Eliminating them, one recovers (1).

The action with Lagrangian (2) is evidently invariant under world-volume diffeomorphisms and abelian gauge

transformations of a_μ . For $p = 1$ it is in addition gauge invariant under Weyl transformations of $\varrho_{\mu\nu}$ and we can decompose the latter according to

$$p = 1 : \varrho_{\mu\nu} = \gamma_{\mu\nu} + \sqrt{\gamma} \epsilon_{\mu\nu} \varphi \quad (3)$$

with

$$\gamma_{\mu\nu} = \gamma_{\nu\mu}, \quad \gamma = -\det(\gamma_{\mu\nu}), \quad \epsilon_{21} = \epsilon^{12} = 1$$

where we assumed for definiteness that $\gamma_{\mu\nu}$ has Lorentzian signature. Since $\sqrt{\gamma} \epsilon_{\mu\nu}$ behaves as a covariant 2-tensor field under world-sheet diffeomorphisms and has the same Weyl weight as $\gamma_{\mu\nu}$, φ transforms as a scalar field under diffeomorphisms and is Weyl invariant. Using (3), the Lagrangian (2) for $p = 1$ reads

$$L_1 = \frac{1}{2} (1 - \varphi^2)^{-1/2} (\sqrt{\gamma} \gamma^{\mu\nu} \mathcal{G}_{\mu\nu} - \varphi \epsilon^{\mu\nu} F_{\mu\nu}). \quad (4)$$

We are now looking for generalizations of (4), guided by its field content and gauge symmetries. Since the gauge symmetries of (4) treat φ on an equal footing with the x^m , we can treat φ as zeroth coordinate of an extended target space with coordinates X^M ,

$$\{X^M\} = \{\varphi, x^m\}, \quad \varphi \equiv X^0.$$

Furthermore we allow for a set of abelian gauge fields a_μ^I rather than only one such gauge field, and fix the field content of the models to be studied to

$$\{\phi^i\} = \{\gamma_{\mu\nu}, a_\mu^I, X^M\}. \quad (5)$$

In addition to this field content, we impose gauge invariance under world-sheet diffeomorphisms, Weyl transformations of the $\gamma_{\mu\nu}$, and abelian gauge transformations of the a_μ^I . Infinitesimally these gauge transformations read

$$\begin{aligned} \delta\gamma_{\mu\nu} &= \varepsilon^\rho \partial_\rho \gamma_{\mu\nu} + 2\gamma_{\rho(\nu} \partial_{\mu)} \varepsilon^\rho + \lambda \gamma_{\mu\nu}, \\ \delta a_\mu^I &= \varepsilon^\nu \partial_\nu a_\mu^I + a_\nu^I \partial_\mu \varepsilon^\nu + \partial_\mu \Lambda^I, \\ \delta X^M &= \varepsilon^\mu \partial_\mu X^M \end{aligned} \quad (6)$$

where ε^μ , λ and Λ^I parametrize world-sheet diffeomorphisms, Weyl transformations and gauge transformations of the a_μ^I respectively. Our first result is that, up to a total derivative, the most general Lagrangian which is (a) constructible solely of the fields (5), (b) local (= polynomial in derivatives of any order), and (c) up to a total derivative invariant under the gauge transformations (6), is

$$\begin{aligned} L &= \frac{1}{2} \sqrt{\gamma} \gamma^{\mu\nu} G_{MN}(X) \partial_\mu X^M \cdot \partial_\nu X^N \\ &+ \epsilon^{\mu\nu} \left[\frac{1}{2} B_{MN}(X) \partial_\mu X^M \cdot \partial_\nu X^N \right. \\ &\left. + D_I(X) \partial_\mu a_\nu^I \right] \end{aligned} \quad (7)$$

where the G_{MN} , B_{MN} and D_I are arbitrary functions of the X^M . This result is the analogue of a similar one holding for bosonic strings [6] and will be proved in [5]. Note that (7) covers in particular D -string actions of a general form, if we choose

$$\begin{aligned} G_{M0} &= 0, \quad G_{mn} = g_{mn}(x) f(\varphi), \\ B_{m0} &= 0, \quad B_{mn} = b_{mn}(x) \sqrt{f^2(\varphi) - 1}, \\ D_I &= d_I(x) \sqrt{f^2(\varphi) - 1} \end{aligned} \quad (8)$$

where $f(\varphi)$ is (almost) arbitrary (this arbitrariness reflects the freedom of field redefinitions $\varphi \rightarrow \tilde{\varphi}(\varphi)$). Indeed, upon elimination of $\gamma_{\mu\nu}$ and φ , (8) yields Born–Infeld actions generalizing (1) among others to curved target spaces:

$$\begin{aligned} S_1 &= \int d^2\sigma \sqrt{-\det(\hat{\mathcal{G}}_{\mu\nu} + \mathcal{F}_{\mu\nu})}, \\ \hat{\mathcal{G}}_{\mu\nu} &= g_{mn}(x) \partial_\mu x^m \cdot \partial_\nu x^n, \\ \mathcal{F}_{\mu\nu} &= d_I(x) (\partial_\mu a_\nu^I - \partial_\nu a_\mu^I) \\ &+ b_{mn}(x) \partial_\mu x^m \cdot \partial_\nu x^n. \end{aligned} \quad (9)$$

We note that (7) covers for instance also actions with Lagrangian

$$L = \sqrt{-\det(\hat{\mathcal{G}}_{\mu\nu} + \mathcal{F}_{\mu\nu})} - \sqrt{-\det(\hat{\mathcal{G}}_{\mu\nu})} \quad (10)$$

which were considered already by Born and Infeld (for $p = 3$) [13] and are obtained analogously by choosing $G_{mn} = g_{mn}(x) [f(\varphi) - 1]$ and the remaining functions as in (8).

RIGID SYMMETRIES

Our second result is that the nontrivial rigid symmetries of an action with Lagrangian (7) are generated by transformations

$$\begin{aligned} \Delta X^M &= \mathcal{X}^M(X), \quad \Delta\gamma_{\mu\nu} = 0, \\ \Delta a_\mu^I &= -\sqrt{\gamma} \epsilon_{\mu\nu} \mathcal{A}_M^I(X) \partial^\nu X^M \\ &+ \mathcal{B}_M^I(X) \partial_\mu X^M + a_\mu^J \mathcal{C}_J^I(X) \end{aligned} \quad (11)$$

with functions $\mathcal{X}^M(X)$, $\mathcal{A}_M^I(X)$, $\mathcal{B}_M^I(X)$, $\mathcal{C}_J^I(X)$ solving

$$\mathcal{L}_\mathcal{X} G_{MN} = -2\mathcal{A}_{(M}^I \partial_{N)} D_I, \quad (12)$$

$$\mathcal{L}_\mathcal{X} B_{MN} = 2\partial_{[N} \mathcal{Y}_{M]} + 2\mathcal{B}_{[N}^I \partial_{M]} D_I, \quad (13)$$

$$\mathcal{L}_\mathcal{X} \partial_M D_I = -\mathcal{C}_I^J \partial_M D_J \quad (14)$$

for some functions $\mathcal{Y}_M(X)$. Here $\mathcal{L}_\mathcal{X}$ denotes the standard Lie derivative along \mathcal{X} , and we used

$$\partial^\mu = \gamma^{\mu\nu} \partial_\nu, \quad \partial_M = \partial/\partial X^M.$$

Note that equations (12–14) generalize the familiar Killing vector equations for the target space. The general form of the latter (with nonvanishing B_{MN}) was discussed in [7,6] and arises from (12–14) for $D_I = 0$ (as $D_I = 0$ reproduces the usual bosonic string, we thus recover for this case the result of [6] for the rigid symmetries of the bosonic string). We will solve these equations explicitly for specific models in the next section. In [5] we will prove that the above symmetries exhaust the nontrivial rigid symmetries of an action with Lagrangian (7)¹. Here we only note that, under transformations (11) satisfying (12–14), the Lagrangian (7) indeed transforms into a total derivative as can be easily verified,

$$\Delta L = \epsilon^{\mu\nu} \partial_\mu (-\mathcal{Y}_M \partial_\nu X^M + a_\nu^I \mathcal{X}^M \partial_M D_I + D_I \Delta a_\nu^I). \quad (15)$$

The conserved Noether currents j^μ corresponding to the symmetries (11) are now readily computed,

$$j^\mu = \sqrt{\gamma} \gamma^{\mu\nu} G_{MN} \mathcal{X}^M \partial_\nu X^N + \epsilon^{\mu\nu} (\tilde{\mathcal{Y}}_M \partial_\nu X^M - a_\nu^I \mathcal{X}^M \partial_M D_I) \quad (16)$$

where

$$\tilde{\mathcal{Y}}_M = \mathcal{Y}_M - B_{MN} \mathcal{X}^N.$$

In order to complete the above statements about the rigid symmetries, we note that the solutions to the generalized Killing vector equations (12–14) are determined only up to the redefinitions

$$\begin{aligned} \mathcal{A}_M^I &\rightarrow \mathcal{A}_M^I + \mathcal{E}^{[IJ]} \partial_M D_J, \\ \mathcal{B}_M^I &\rightarrow \mathcal{B}_M^I + \partial_M \mathcal{B}^I + \mathcal{E}^{(IJ)} \partial_M D_J, \\ \mathcal{Y}_M &\rightarrow \mathcal{Y}_M + \partial_M \mathcal{Y} - \mathcal{B}^I \partial_M D_I \end{aligned} \quad (17)$$

where the $\mathcal{B}^I(X)$, $\mathcal{E}^{IJ}(X)$ and $\mathcal{Y}(X)$ are arbitrary functions of the X^M . These redefinitions drop out of (12–14) and affect in (11) only the transformations of the a_μ^I according to

$$\begin{aligned} \Delta a_\mu^I &\rightarrow \Delta a_\mu^I + \partial_\mu \mathcal{B}^I + \mathcal{E}_{\mu\nu}^{IJ} \epsilon^{\nu\rho} \partial_\rho D_J, \\ \mathcal{E}_{\mu\nu}^{IJ} &= -\sqrt{1/\gamma} \gamma_{\mu\nu} \mathcal{E}^{[IJ]} + \epsilon_{\mu\nu} \mathcal{E}^{(IJ)}. \end{aligned} \quad (18)$$

These are irrelevant redefinitions of the rigid symmetries, i.e. two rigid symmetries are identified if they coincide up to such redefinitions. Namely $\partial_\mu \mathcal{B}^I$ are just special gauge transformations, while $\mathcal{E}_{\mu\nu}^{IJ} \epsilon^{\nu\rho} \partial_\rho D_J$ are on-shell trivial symmetries. The latter holds due to $\mathcal{E}_{\mu\nu}^{IJ} = -\mathcal{E}_{\nu\mu}^{JI}$ and $\epsilon^{\nu\rho} \partial_\rho D_J = \delta S / \delta a_\nu^J$ where S denotes the action with Lagrangian (7).

It is easy to check that the commutator of two symmetries (11) is again a symmetry of this type,

$$[\Delta_1, \Delta_2] = \Delta_3. \quad (19)$$

Namely, using (11) and the notation $\Delta_i X^M = \mathcal{X}_i^M$ etc. ($i=1,2,3$), a direct computation of $[\Delta_1, \Delta_2]$ yields

$$\begin{aligned} \mathcal{X}_3^M &= \mathcal{X}_1^N \partial_N \mathcal{X}_2^M - (1 \leftrightarrow 2), \\ \mathcal{A}_{3M}^I &= \mathcal{L}_{\mathcal{X}_1} \mathcal{A}_{2M}^I - \mathcal{C}_{1J}^I \mathcal{A}_{2M}^J - (1 \leftrightarrow 2), \\ \mathcal{B}_{3M}^I &= \mathcal{L}_{\mathcal{X}_1} \mathcal{B}_{2M}^I - \mathcal{C}_{1J}^I \mathcal{B}_{2M}^J - (1 \leftrightarrow 2), \\ \mathcal{C}_{3J}^I &= \mathcal{L}_{\mathcal{X}_1} \mathcal{C}_{2J}^I - (1 \leftrightarrow 2). \end{aligned} \quad (20)$$

Using standard properties of Lie derivatives such as $[\mathcal{L}_{\mathcal{X}_1}, \mathcal{L}_{\mathcal{X}_2}] = \mathcal{L}_{\mathcal{X}_3}$, it is easy to verify that the set of functions (20) solves (12–14) with $\mathcal{Y}_{3M} = \mathcal{L}_{\mathcal{X}_1} \mathcal{Y}_{2M} - \mathcal{L}_{\mathcal{X}_2} \mathcal{Y}_{1M}$ whenever the sets $(\mathcal{X}_i^M, \mathcal{A}_{iM}^I, \mathcal{B}_{iM}^I, \mathcal{C}_{iJ}^I, \mathcal{Y}_{iM})$, $i = 1, 2$ solve (12–14) too. In that sense, the commutators of symmetry transformations (11) ‘close’. However, this does *not* necessarily imply that the algebra of the rigid symmetries closes off-shell in a particular basis of the rigid symmetries. Namely suppose that Δ_1 and Δ_2 are two elements of such a basis. Their commutator (19) will in general be a linear combination of elements of the basis only up to redefinitions (18). Hence, in general the algebra of the elements of the basis will close only up to gauge transformations and on-shell trivial transformations of the type occurring in (18).

To summarize, the nontrivial (infinitesimal) rigid symmetries of an action with Lagrangian (7) are exhausted by transformations (11) with target space functions satisfying (12–14), and defined modulo the redefinitions (17). Furthermore, any solution to (12–14) which does not vanish modulo redefinitions (17) gives rise to a nontrivial rigid symmetry generated by (11). Hence, one has precisely to solve (12–14) in order to find all rigid symmetries of an action with Lagrangian (7). A basis of the rigid symmetries is obtained from a basis of solutions to (12–14), i.e. from a complete set of solutions which are linearly independent up to redefinitions (17). Needless to say that, on general grounds, rigid symmetries of Born-Infeld actions (9) arise from those of the corresponding actions with Lagrangians (7,8) by replacing the auxiliary fields $\gamma_{\mu\nu}$ and φ in Δx^m and Δa_μ^I with a solution to their algebraic equations of motion, i.e. by substituting for instance

$$\begin{aligned} f(\varphi) &\rightarrow \sqrt{\det(\hat{\mathcal{G}}_{\mu\nu}) / \det(\hat{\mathcal{G}}_{\mu\nu} + \mathcal{F}_{\mu\nu})}, \\ \gamma_{\mu\nu} &\rightarrow \hat{\mathcal{G}}_{\mu\nu}. \end{aligned} \quad (21)$$

Note that $\gamma_{\mu\nu}$ is actually defined by the equations of motion only up to a completely arbitrary function multiplying $\hat{\mathcal{G}}_{\mu\nu}$ due to the Weyl invariance of (7). As this general function drops out of the transformations (11), one can indeed choose (21) with no loss of generality.

¹The rigid symmetries are obtained from the BRST cohomology at ghost number -1 [14].

EXAMPLES AND DISCUSSION

To illustrate and interpret the general results presented above, we will now solve (12–14) explicitly for a specific class of models and discuss the corresponding rigid symmetries. The models are characterized by Lagrangians (7) involving only one $U(1)$ gauge field a_μ , with

$$\begin{aligned} G_{0M} &= 0, \quad G_{mn} = f(\varphi) \eta_{mn}, \\ B_{0m} &= 0, \quad B_{mn} = B_{mn}(\varphi), \quad D = D(\varphi). \end{aligned} \quad (22)$$

Recall that the special choice $B_{mn} = 0$, $D = (f^2 - 1)^{1/2}$ reproduces the action (1) for $p = 1$. We will first show that, for any choice of f and $D \neq \text{constant}$, the general solution of eqs. (12–14) is, up to redefinitions (17),

$$\begin{aligned} \mathcal{X}^0 &= \mathcal{X}^0(\varphi), \\ \mathcal{X}^m &= -(f'/2f) \mathcal{X}^0 x^m \\ &\quad + a^m(\varphi) + a^{mn}(\varphi) x_n, \quad a^{mn} = -a^{nm}, \\ \mathcal{A}_m &= -\eta_{mn}(f/D')(\mathcal{X}^n)', \quad \mathcal{A}_0 = 0, \\ \mathcal{B}_m &= (1/D')[B'_{mn} \mathcal{X}^n + \frac{1}{2}(B'_{mn} \mathcal{X}^0)' x^n], \quad \mathcal{B}_0 = 0, \\ \mathcal{C} &= -(\mathcal{X}^0 D')'/D', \\ \mathcal{Y}_m &= \frac{1}{2} B'_{mn} \mathcal{X}^0 x^n + B_{mn} \mathcal{X}^n, \quad \mathcal{Y}_0 = 0, \end{aligned} \quad (23)$$

where $\mathcal{X}^0(\varphi)$, $a^m(\varphi)$ and $a^{mn}(\varphi)$ are *arbitrary functions* of φ and we used

$$' \equiv \partial/\partial\varphi, \quad x_m \equiv \eta_{mn} x^n.$$

Let us now sketch the derivation of (23). The results for \mathcal{X}^0 and \mathcal{C} follow immediately from (14) as it reads in the cases under study $D' \partial_m \mathcal{X}^0 = 0$ for $M = m$, and $(\mathcal{X}^0 D')' = -\mathcal{C} D'$ for $M = 0$. The remaining results follow from (12) and (13). To show this, we regard (12) and (13), for any fixed function $\mathcal{X}^0(\varphi)$, as a set of inhomogeneous equations for the \mathcal{X}^m , \mathcal{A}_M , \mathcal{B}_M and \mathcal{Y}_M . The general solution is then the sum of a particular solution and the general solution of the homogeneous equations. A particular solution is given by

$$\mathcal{X}^{(p)m} = -(f'/2f) \mathcal{X}^0 x^m \quad (24)$$

and corresponding expressions for $\mathcal{A}_M^{(p)}$, $\mathcal{B}_M^{(p)}$ and $\mathcal{Y}_M^{(p)}$ obtained from (23) for $\mathcal{X}^m \rightarrow \mathcal{X}^{(p)m}$. The homogeneous equations (12) and (13), obtained by setting $\mathcal{X}^0 = 0$, yield, for $(M, N) = (m, n)$, $(M, N) = (m, 0)$ and $(M, N) = (0, 0)$ respectively,

$$\eta_{nk} \partial_m \mathcal{X}^{(h)k} + \eta_{mk} \partial_n \mathcal{X}^{(h)k} = 0, \quad (25)$$

$$\eta_{mn} (\mathcal{X}^{(h)n})' f = -\mathcal{A}_m^{(h)} D', \quad 0 = \mathcal{A}_0^{(h)} D', \quad (26)$$

$$\partial_n \tilde{\mathcal{Y}}_m^{(h)} - \partial_m \tilde{\mathcal{Y}}_n^{(h)} = 0, \quad (27)$$

$$(\tilde{\mathcal{Y}}_m^{(h)})' - \partial_m \mathcal{Y}_0^{(h)} - \mathcal{B}_m^{(h)} D' + B'_{mn} \mathcal{X}^{(h)n} = 0 \quad (28)$$

where we used that B_{mn} depends only on φ and defined

$$\tilde{\mathcal{Y}}_m^{(h)} = \mathcal{Y}_m^{(h)} - B_{mn} \mathcal{X}^{(h)n}.$$

(25) are just the Killing vector equations for a flat space with coordinates x^m and thus have the general solution

$$\mathcal{X}^{(h)m} = a^m(\varphi) + a^{mn}(\varphi) x_n, \quad a^{mn} = -a^{nm}. \quad (29)$$

(26) can be solved for the $\mathcal{A}_M^{(h)}$ and thus determines directly these functions. (27) implies $\tilde{\mathcal{Y}}_m^{(h)} = \partial_m \mathcal{Y}$ for some $\mathcal{Y}(X)$. Using this in (28), we get $\mathcal{B}_m^{(h)} = \partial_m \mathcal{B} + B'_{mn} \mathcal{X}^{(h)n}/D'$ with $\mathcal{B} = (\mathcal{Y}' - \mathcal{Y}_0^{(h)})/D'$, and thus also $\mathcal{Y}_0^{(h)} = \mathcal{Y}' - \mathcal{B} D'$. Furthermore, we have the trivial identity $\mathcal{B}_0^{(h)} = \mathcal{B}' + \mathcal{E} D'$ with $\mathcal{E} = (\mathcal{B}_0^{(h)} - \mathcal{B}')/D'$. Now, contributions $\partial_M \mathcal{Y} - \mathcal{B} \partial_M D$ and $\partial_M \mathcal{B} + \mathcal{E} \partial_M D$ to \mathcal{Y}_M and \mathcal{B}_M respectively can be removed by redefinitions (17). Without loss of generality, we can thus choose

$$\begin{aligned} \mathcal{Y}_m^{(h)} &= B_{mn} \mathcal{X}^{(h)n}, \quad \mathcal{Y}_0^{(h)} = 0, \\ \mathcal{B}_m^{(h)} &= B'_{mn} \mathcal{X}^{(h)n}/D', \quad \mathcal{B}_0^{(h)} = 0. \end{aligned} \quad (30)$$

Altogether this yields (23).

Let us now discuss the symmetries (11) arising from (23). As they involve arbitrary functions $\mathcal{X}^0(\varphi)$, $a^m(\varphi)$ and $a^{mn}(\varphi)$, we conclude immediately that any model characterized by (22) possesses *infinitely many nontrivial rigid symmetries*. To interpret them, we will use the equations of motion, as rigid symmetries map in general solutions to the equations of motion to other solutions. The equations of motion for $\gamma_{\mu\nu}$ are solved for instance by $\gamma_{\mu\nu} = \mathcal{G}_{\mu\nu}$ with $\mathcal{G}_{\mu\nu}$ as in (1). The equation of motion for a_μ yields

$$\epsilon^{\mu\nu} \partial_\nu D(\varphi) = 0 \Rightarrow \varphi = \varphi_0 = \text{constant} \quad (31)$$

i.e. φ is on-shell just a constant fixed by initial conditions. The value of this constant distinguishes thus partly different solutions. Furthermore it controls among others the coupling of the gauge field to the string, as the equations of motion for φ and x^m yield respectively

$$\epsilon^{\mu\nu} (F_{\mu\nu} + \mathcal{B}_{\mu\nu}) = K(\varphi_0) \sqrt{\mathcal{G}}, \quad (32)$$

$$\partial_\mu (\sqrt{\mathcal{G}} \mathcal{G}^{\mu\nu} \partial_\nu x_m + \tilde{B}_{mn}(\varphi_0) \epsilon^{\mu\nu} \partial_\nu x^n) = 0 \quad (33)$$

where we have defined

$$\begin{aligned} \mathcal{G} &= -\det(\mathcal{G}_{\mu\nu}), \\ \mathcal{B}_{\mu\nu} &= B_{mn}(\varphi_0) \partial_\mu x^m \cdot \partial_\nu x^n, \\ \tilde{B}_{mn}(\varphi_0) &= B_{mn}(\varphi_0) D(\varphi_0)/f(\varphi_0), \\ K(\varphi_0) &= -2f'(\varphi_0)/D'(\varphi_0). \end{aligned}$$

Note that (33) are nothing but the equations of motion for an ordinary bosonic string with constant \tilde{B}_{mn} and that (32) relates the abelian gauge field to this string. Now, Δx^m reads

$$\Delta x^m = a(\varphi) x^m + a^m(\varphi) + a^{mn}(\varphi) x_n \quad (34)$$

where $a = -\mathcal{X}^0 f'/2f$. As φ is constant for any solution of the equations of motion, (34) generates on-shell Poincaré transformations and dilatations² of the target space coordinates. The important property of the new symmetries is that they transform in addition the gauge field a_μ nontrivially. In particular, for a transformation (34) which generates on-shell a dilatation of x^m , we get in the case $B_{mn} = 0$:

$$\begin{aligned} B_{mn} = 0, \quad \Delta x^m &= a(\varphi) x^m \Rightarrow \\ \Delta a_\mu &= \epsilon_{\mu\nu} \mathcal{A}(\varphi) \sqrt{g} g^{\nu\rho} x^m \partial_\rho x_m + \mathcal{C}(\varphi) a_\mu \end{aligned} \quad (35)$$

where $\mathcal{A} = a'f/D'$ and $\mathcal{C} = (2afD'/f')/D'$. Now, even for $a_\mu = 0$, (35) does in general not reduce to a gauge transformation (not even on-shell!). In particular, it maps thus in general a solution to the equations of motion with $a_\mu = 0$ to another solution with nonvanishing field strength $F_{\mu\nu}$. Indeed, (32) shows that solutions with vanishing $F_{\mu\nu}$ correspond in the case $B_{mn} = 0$ to special values of φ_0 , namely roots of the function K ,

$$B_{mn} = 0 : F_{\mu\nu} = 0 \leftrightarrow K(\varphi_0) = 0.$$

As transformations (35) are accompanied by transformations $\Delta\varphi = \mathcal{X}^0(\varphi)$ (recall that $a = -\mathcal{X}^0 f'/2f$), we conclude: *in models with $B_{mn} = 0$, any transformation $\Delta\varphi = \mathcal{X}^0(\varphi)$ which changes the value of $K(\varphi_0)$ from 0 to a nonvanishing one, is accompanied by a transformation $F_{\mu\nu} = 0 \rightarrow F_{\mu\nu} \neq 0$!* Completely analogous considerations apply of course to models with $B_{mn} \neq 0$.

Let us now discuss the off-shell algebra of the symmetries arising from (23). As the transformations (34) can be regarded as φ -dependent Poincaré transformations and dilatations of the target space coordinates, their algebra will in any basis be a Kač–Moody version of the Weyl algebra. A basis is obtained by choosing a suitable basis for the functions $\mathcal{X}^0(\varphi)$, $a^m(\varphi)$ and $a^{mn}(\varphi)$ occurring in (23), adapted to the properties (e.g. boundary conditions, topology) of the specific model one wants to study. To give an explicit example, we consider the case

$$f(\varphi) = \exp(\varphi)$$

and functions $\mathcal{X}^0(\varphi)$, $a^m(\varphi)$ and $a^{mn}(\varphi)$ which can be expanded in integer powers of $\exp(\varphi)$, i.e.

$$\begin{aligned} \mathcal{X}^0(\varphi) &= c_a e^{-a\varphi}, \\ a^m(\varphi) &= c_a^m e^{-a\varphi}, \quad a^{mn}(\varphi) = c_a^{mn} e^{-a\varphi} \end{aligned}$$

where the c 's are constant infinitesimal transformation parameters indexed by $a \in \mathbf{Z}$, and summation over a is understood. We now decompose Δ according to

$$\Delta = c_a L^a + c_a^m P_m^a + \frac{1}{2} c_a^{mn} M_{mn}^a$$

where L^a , P_m^a and $M_{mn}^a = -M_{nm}^a$ are the generators of rigid symmetries, the algebra of which we want to compute. (11) and (23) yield

$$\begin{aligned} L^a \varphi &= e^{-a\varphi}, \quad L^a x^m = -\frac{1}{2} e^{-a\varphi} x^m, \\ P_m^a \varphi &= 0, \quad P_m^a x^n = e^{-a\varphi} \delta_m^n, \\ M_{mn}^a \varphi &= 0, \quad M_{mn}^a x^k = e^{-a\varphi} (\delta_m^k x_n - \delta_n^k x_m). \end{aligned}$$

Analogously one determines readily the transformations of a_μ . For instance, one gets

$$\begin{aligned} L^a a_\mu &= \mathcal{Z}^a(\varphi) \sqrt{\gamma} \epsilon_{\mu\nu} \partial^\nu (x_m x^m) \\ &\quad + \mathcal{Z}_{mn}^a(\varphi) x^n \partial_\mu x^m + \mathcal{C}^a(\varphi) a_\mu \end{aligned}$$

where

$$\begin{aligned} \mathcal{Z}^a &= a e^{(1-a)\varphi} / (4D'), \\ \mathcal{Z}_{mn}^a &= [B_{mn}'' - (1+a)B_{mn}'] e^{-a\varphi} / (2D'), \\ \mathcal{C}^a &= e^{-a\varphi} (a - D''/D'). \end{aligned}$$

It is now very easy to compute the symmetry algebra on the X^M . On the gauge field it is more involved, but by means of the general arguments given in the previous section one concludes that the algebra coincides necessarily on all fields up to gauge transformations and on-shell trivial symmetries of the type occurring in (18). If the latter are present, the algebra is open. This turns out to be the case in general. However, at least for $B_{mn} = 0$ the algebra closes off-shell even on a_μ and reads

$$\begin{aligned} [L^a, L^b] &= (a-b) L^{a+b}, \\ [L^a, P_m^b] &= \left(\frac{1}{2} - b\right) P_m^{a+b}, \\ [L^a, M_{mn}^b] &= -b M_{mn}^{a+b}, \\ [P_m^a, P_n^b] &= 0, \\ [M_{mn}^a, P_k^b] &= \eta_{km} P_n^{a+b} - (m \leftrightarrow n), \\ [M_{mn}^a, M_{pq}^b] &= 2\eta_{p[m} M_{n]q}^{a+b} - (p \leftrightarrow q). \end{aligned}$$

This is what we call a Kač–Moody version of the Weyl algebra.

Let us briefly point out an immediate generalization of the above results to models characterized by

$$\begin{aligned} G_{0M} &= 0, \quad G_{mn} = f(\varphi) g_{mn}(x), \\ B_{0m} &= 0, \quad B_{mn} = h(\varphi) b_{mn}(x), \quad D = D(\varphi), \end{aligned} \quad (36)$$

i.e. we allow now of curved target space metrics $g_{mn}(x)$ and x -dependent B_{mn} . Suppose that $\{\zeta_A^m(x), \mathcal{Y}_{mA}(x)\}$ is a basis of inequivalent solutions to

²These dilatational symmetries should neither be confused with the world-sheet Weyl invariance of (4) and the world-volume Weyl invariance of certain formulations of the (super) p -brane, nor with the linearly realized global target space scale invariance of the formulation [15] treating the string tension of a p -brane as a dynamical variable.

$$\mathcal{L}_{\zeta_A} g_{mn} = 0, \quad \mathcal{L}_{\zeta_A} b_{mn} = 2\partial_{[n}\mathcal{Y}_{m]A} \quad (37)$$

where two solutions are called equivalent if they differ only by $\mathcal{Y}_m \rightarrow \mathcal{Y}_m + \partial_m \mathcal{Y}$. Then solutions to (12–14) are given by

$$\begin{aligned} \mathcal{X}^m &= c^A(\varphi) \zeta_A^m(x), \quad \mathcal{X}^0 = 0, \\ \mathcal{Y}_m &= c^A(\varphi) \mathcal{Y}_{mA}(x), \quad \mathcal{Y}_0 = 0, \\ \mathcal{A}_m &= -(f/D') g_{mn} (\mathcal{X}^n)', \quad \mathcal{A}_0 = 0, \\ \mathcal{B}_m &= (h/D') (\mathcal{Y}_m - b_{mn} \mathcal{X}^n)', \quad \mathcal{B}_0 = \mathcal{C} = 0 \end{aligned} \quad (38)$$

where $c^A(\varphi)$ are arbitrary functions. We conclude that any Killing vector field $\zeta_A(x)$ of the target space satisfying (37) gives rise to infinitely many rigid symmetries of the model characterized by (36). The algebra of these symmetries is a Kač–Moody version of the Lie algebra of the \mathcal{L}_{ζ_A} and generalizes the Poincaré Kač–Moody algebra found above. It appears to depend on g_{mn} and b_{mn} whether there is a generalization of the dilatational symmetries too.

SUPERSYMMETRIC EXTENSIONS

One might wonder whether there are supersymmetric extensions of the symmetry structure presented above. We have not investigated this question in detail by means of a cohomological analysis. However we have found such extensions in simple cases. For instance, consider the Lagrangian

$$L = \frac{1}{2} f(\varphi) \sqrt{\gamma} \gamma^{\mu\nu} \Pi_\mu^m \Pi_\nu^n \eta_{mn} + D(\varphi) \epsilon^{\mu\nu} \partial_\mu a_\nu \quad (39)$$

where, as in (22), f and $D \neq \text{constant}$ are any functions of φ , and, using the conventions and notation of [2],

$$\Pi_\mu^m = \partial_\mu x^m - \bar{\theta} \Gamma^m \partial_\mu \theta. \quad (40)$$

The action with Lagrangian (39) is invariant under the following rigid supersymmetry transformations

$$\begin{aligned} Q\theta^\alpha &= c^\alpha B(\varphi), \\ Qx^m &= \bar{c} \Gamma^m \theta B(\varphi), \\ Qa_\mu &= 2\bar{c} \Gamma^m \theta \sqrt{\gamma} \epsilon_{\mu\nu} \Pi_m^\nu B'(\varphi) f(\varphi)/D'(\varphi), \\ Q\varphi &= Q\gamma_{\mu\nu} = 0 \end{aligned} \quad (41)$$

where c^α is a constant anticommuting target space spinor, $B(\varphi)$ is an arbitrary function of φ , and

$$\Pi_m^\mu = \eta_{mn} \gamma^{\mu\nu} \Pi_\nu^n.$$

The commutator of two transformations (41) reads

$$[Q_1, Q_2] = 2\bar{c}_2 \Gamma^m c_1 P_m \quad (42)$$

where P_m generates φ -dependent “translations” of the type found above in the nonsupersymmetric case,

$$\begin{aligned} P_m x^n &= \delta_m^n B_{12}(\varphi), \\ P_m a_\mu &= \sqrt{\gamma} \epsilon_{\mu\nu} \Pi_m^\nu B'_{12}(\varphi) f(\varphi)/D'(\varphi), \\ P_m \varphi &= P_m \gamma_{\mu\nu} = P_m \theta^\alpha = 0 \end{aligned} \quad (43)$$

with

$$B_{12}(\varphi) = B_1(\varphi) B_2(\varphi).$$

Together with the analogues of the symmetries arising from (11), the supersymmetries (41) form a Kač–Moody super-Weyl algebra which can be easily constructed explicitly along the lines of the previous section.

CONCLUSION

Any local action in two dimensions ($p = 1$) with field content and gauge symmetries given by (5) and (6) respectively, has a Lagrangian of the form (7). Specific choices of G_{MN} , B_{MN} and D_I provide actions which turn upon elimination of the auxiliary fields into D -string actions of the Born–Infeld type such as (1) (for $p = 1$) or, more generally, (9) or (10).

The rigid symmetries of an action with Lagrangian (7) are determined by the solutions of the generalized Killing vector equations (12–14). We have shown that these equations can have infinitely many inequivalent solutions which we have spelled out explicitly for specific models in a flat target space. In the latter models, we have found a Kač–Moody realization of the Weyl group, the new symmetries being non-linearly realized. Symmetries of the actions (1) (for $p = 1$), (9) and (10) are obtained from those of their counterparts (7) simply by eliminating the auxiliary fields. For instance, from (35) one obtains in this way among others a symmetry of the $p = 1$ -action (1) generated by

$$\begin{aligned} \Delta x^m &= F x^m, \\ \Delta a_\mu &= \epsilon_{\mu\nu} (F^2 - 1) \sqrt{\mathcal{G}} \mathcal{G}^{\nu\rho} x^m \partial_\rho x_m + 2F a_\mu \end{aligned} \quad (44)$$

where, assuming that $\mathcal{G}_{\mu\nu}$ has Lorentzian signature,

$$F = \mathcal{G}^{-1/2} \epsilon^{\mu\nu} \partial_\mu a_\nu, \quad \mathcal{G} = -\det(\mathcal{G}_{\mu\nu}).$$

F is constant on-shell. The value of this constant characterizes partly a solution to the equations of motion and contributes to its string tension. (44) generates on-shell a dilatation of the target space coordinates, but it also transforms the abelian gauge field nontrivially. In particular, it transforms a solution to the equations of motion with $F = 0$ to another one with $F \neq 0$, as on-shell one has $\Delta F = 2(F^2 - 1)$. Symmetries such as (44) are thus useful, among others, to connect configurations of the fundamental string with those of the D-string.

We have also shown that the Kač–Moody symmetry structure extends analogously to curved target spaces

if the latter possess Killing vector fields satisfying (37). Furthermore we have given some examples of supersymmetric extensions where an infinite number of rigid supersymmetries appears in addition to the Kač–Moody–Weyl symmetries.

We have obtained our results by an analysis of the BRST cohomology at ghost numbers 0 (actions) and -1 (symmetries). Especially the results on the rigid symmetries are difficult to guess or to derive by other means due to highly nonlinear nature of symmetries such as (44).

One may speculate whether the infinite number of symmetries reflects part of the space-time symmetry structure of an underlying (“M”) theory. This possibility is suggested because D -branes may probe shorter space-time distances than strings [16]. It would be interesting to further understand the physical meaning of the Weyl–Kač–Moody algebra and whether or not the κ -invariant formulation of the D -string has infinitely many rigid symmetries too. Another interesting point to be investigated will be to check whether a Weyl–Kač–Moody algebra appears also for other D - p -branes.

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